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Large gain, negative resistance, and oscillations in superconducting quasiparticle heterodyne mixers

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We have measured the properties of a superconductor-insulator-superconductor quasiparticle mixer which is operated in the quantum limit. Single sideband conversion gain larger than +4 dB was observed at 36 GHz with a mixer noise temperature $T = 9 \pm 6$ K, which is to be compared with the (Planck) quantum limit $\hbar \omega / (k \ln 2) \approx 2.5$ K. Complete three-port mixer calculations are presented which are in good agreement with the gain measurements. The dynamic resistance was observed to become infinite and then negative as the source conductance was decreased. This implies that arbitrarily large gain is available. The negative resistance is accompanied by IF oscillations.

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Low-noise microwave heterodyne receivers which use a superconductor-insulator-superconductor (SIS) tunnel junction in the quasiparticle regime for the mixing element are being actively investigated. Mixing has been reported over the frequency range from 10 to 115 GHz in lead-alloy tunnel junctions. Arrays of junctions in series have been successfully used in addition to single junctions. The conversion efficiencies measured at 36 and 74 GHz are larger than the limit obtainable from resistive mixers. The mixer noise temperature measured at 36 GHz is comparable with the quantum limit $T_Q \approx \hbar \omega / k \ln 2$.

The photon assisted tunneling theory of quasiparticle microwave mixers has been explored extensively. Calculations predict that there is a wide range of experimental parameter values for which SIS mixers will have large conversion gain and noise comparable to the quantum limit. The importance of conversion gain is evident from the equation for the noise temperature $T_R$ of a heterodyne receiver with a coupled antenna noise $T_A$,

$$T_R = T_A + T_M + T_{IF} / G,$$

where the noise temperature $T_{IF}$ of the intermediate frequency amplifier is reduced by the mixer conversion gain $G$. We describe here the first observation of coupled IF output power from a quasiparticle mixer which is larger than the available input signal. The values of rf source impedance and LO power at which this gain is observed are in good agreement with the results of three-port mixer calculations.

In calculations of SIS mixer performance large gain is accompanied by large values and even negative values of dynamic resistance $R_D$ at the IF frequency. Although negative resistance due to Josephson pair tunneling is commonplace, rf-induced negative quasiparticle resistance is a new phenomenon. We have observed arbitrarily large (and negative) values of $R_D$ which implies that arbitrarily large gain is available.

For our experiments we have made tin-tin oxide-tin SIS junctions on Si substrates using photore sist bridge masks. Typical junctions have areas of $\sim 10 \mu m^2$ and critical current densities of $\sim 250$ A cm$^{-2}$. The tunneling $I-V$ curves, shown in Fig. 1, are very sharp. Using these junctions we are able to observe strong quantum effects at an operating frequency low enough that accurate microwave measurements are relatively easy.

The apparatus used for mixer measurements is the same as that described in previous publications. The helium
bath temperature was generally 1.5 K, although mixer efficiency varied only weakly with temperature for $T \leq T_{c}/2$. The Sn junction was placed in the E-field direction across a Ka-band waveguide. Radio-frequency matching was obtained by adjusting a screw tuner and a sliding backshort. Room-temperature radiation was reduced by a cold 18-dB attenuator in front of the mixer. The signal and LO power were obtained from carefully calibrated 36-GHz oscillators. Conversion efficiency measurements were made with an IF frequency of 50 MHz and noise measurements with an IF band from 15 to 130 MHz.

In Fig. 1(a) we show a series of measured $I$-$V$ curves for a 22-$\Omega$ junction for values of rf source impedance in the region of optimum mixer operation. A slight increase in rf source impedance produced $I$-$V$ curves with both larger values of dynamic resistance and also negative resistance on the photon assisted tunneling steps as is shown in Fig. 1(b). The region to the left of the dashed line in each part of the figure is very sensitive to an applied magnetic field large enough to produce one flux quantum in the junction. The complicated structure seen there, including regions of negative resistance, arises primarily from pair tunneling effects. The region to the right of the dashed line, on the other hand, is essentially independent of field. We believe that the negative resistance in this latter region arises from the quasiparticle current. In a separate publication we show that this negative resistance can be obtained from a simple analytic model for a pumped junction in the quantum limit with finite rf source resistance.\textsuperscript{11} A magnified view of a negative resistance region measured with a 130-$\Omega$ dc bias resistance is shown in the inset of Fig. 1(b). Oscillations were observed at harmonics of $\sim 5$ MHz when the junction was biased in a negative resistance region. Such oscillations are expected from the interaction of a negative resistance with the bias circuit and thus provide independent evidence for the negative resistance.

Substantial mixer gain was observed by using $I$-$V$ curves similar to those shown in Fig. 1(a). The best experimental values are $+4.3 \pm 1$ dB for the upper sideband and $+3.4 \pm 1$ dB for the lower sideband. The experimental curves from which these results were derived are shown in Fig. 2. The values of conversion efficiency were obtained by dividing the output power coupled to the IF amplifier by the monochromatic signal power (which was $<1.5$ pW) entering the cryostat. Corrections were then made for the measured attenuation of the waveguide ($-1.9$ dB), the cold attenuator ($-17.8$ dB), and the IF cable ($-0.1$ dB). No corrections were made for power lost due to rf or IF mismatch. Significant gain was also observed in the negative resistance regime shown in Fig. 1(b).

To model mixer operation, we propose the equivalent IF output circuit show as an inset in Fig. 3. The mixing action produces a current amplitude $I_{IF}$, which is shunted by the dynamic resistance of the junction $R_{D}$ and the IF amplifier impedance $R_{L}$. For a range of parameters around the maximum gain point, $I_{IF}$ is relatively independent of $R_{D}$ and $V_{dc}$. Theoretical support for this model has been published previously.\textsuperscript{5,14} For matched output ($R_{D} = R_{L}$), the available output power is $P_{RF} \approx R_{D}/8$. In our experiment the output was not matched, so the coupled power is the product of the available power with the mismatch factor $4R_{D}/R_{L}(R_{D} + R_{L})^{3}$. In order to test this model, the coupled IF output power was measured by varying $V_{dc}$ along a single step and was plotted as crosses in Fig. 3 as a function of the
measured $R_D$. The coupled gain numbers were then divided by the mismatch factor to obtain values of the available power shown as circles in Fig. 3. Good agreement is obtained with our assumption that the available output power is proportional to $R_D$. Since the dynamic resistance can be infinite, this model predicts that gain is limited only by the load resistance $R_L$. An improvement of $-1$ dB in measured gain was obtained by using a capacitively loaded length of transmission line to increase $R_L$ above the 50-$\Omega$ input resistance of the IF amplifier.

To determine the mixer noise temperature, the noise contribution of the IF amplifier and of the lossy coaxial IF cable were measured in separate experiments. Subtracting these IF noise sources left the mixer output noise, which was converted into an equivalent input noise by dividing by the SSB gain for the upper sideband. Finally, blackbody radiation $T_A$ present in the waveguide was subtracted leaving the intrinsic mixer input noise temperature of $9 \pm 6$ K. The noise was measured under the operating conditions for maximum gain.

Mixer performance depends critically on the rf source embedding admittance $Y_S$. The experimental value of $Y_S$ can be determined from a pumped $I$-$V$ curve. For our junctions the estimated relaxation parameter $\omega R_N C \approx 7$ at 36 GHz. Therefore, the junction capacitance provides a very low impedance termination at harmonic frequencies, and we can use the three-port $Y$-mixer model. Estimates for $Y_S$ were obtained both by the published method and also by computing a pumped $I$-$V$ curve as a function of source admittance and obtaining the source parameters from a least-squares fit to the corresponding experimental curve. The resulting source admittance is $Y_S = (0.07 \pm 0.01) + j(0.01 \pm 0.02)$ mho. In this fitting the geometrical capacitance of the junction, which contributes

$Y = 0.3$ mho, is regarded as part of the source. Since the deduced value of $Y_S$ is nearly real, it is clear that the matching introduced reactive elements which resonated out much of this junction capacitance at the signal frequency. The linear portion of the quantum reactance of the junction corresponds to $Y \sim 0.01$ mho, so is not quantitatively important.

Mismatch at the IF amplifier also strongly influenced our measured values of gain. For the purposes of these comparisons we define gain in terms of the IF power coupled into a 50-$\Omega$ load. In Fig. 4 we show calculated gain contours for a bias voltage $V = (2\Delta - \hbar\omega/2)/e$ on the first photon assisted tunneling step below $2\Delta/e$. The contours are plotted as a function of normalized rf source conductance $G_s$ and normalized $P_{10}$ so as to show how the gain depends upon these important experimental parameters. It is assumed that all linear rf reactances have been tuned out. Qualitative differences between Fig. 4 and previously published gain contours are due to our use of a fixed value of $R_L$. In addition to the gain contours in Fig. 4 we show the values of the independent variables for which large gain was actually observed. The agreement with theory is quite good. However, there is a significantly discrepancy in the amount of gain which is predicted. It is to be expected that the three-port model overestimates the gain, and that inclusion of dissipation at the harmonic frequencies would improve the agreement between experiment and theory.

When one of our junctions is operated as a direct detector, it is a microwave photodiode with current responsivity $e/\hbar\omega$. The direct detection response can thus be used as a check on the calibration of the rf sources used for mixer tests. The calibration obtained in this way corresponds to less gain and less noise than the direct calibration. This uncertainty is included in the error limits given for the values of gain and noise reported above. We have not accepted the direct detection responsivity as the primary calibration because the possibility exists of gain associated with quasiparticle injection. Because of the small leakage currents in our

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FIG. 3. Equivalent output circuit of an SIS mixer is shown in the inset. The mixer is modeled as a current generator $I_A$ shunted by the dynamic resistance $R_D$ of the pumped junction. The IF amplifier is represented by the load resistance $R_L$. The conversion gain of a fundamental mixer measured for various values of dc bias on the first photon peak below $2\Delta/e$ is shown as a function of the measured $R_D$. The coupled $G_{SB}$ values (crosses) saturate for large values of $R_D$ because of IF impedance mismatch. When corrected for mismatch (circles), the gain is nearly proportional to $R_D$, in agreement with the equivalent circuit for constant $I_A$.

FIG. 4. Gain predicted from a three-port quantum $Y$-mixer model using the experimental $I$-$V$ curve. The junction is based on the first photon assisted tunneling step below $2\Delta/e$. Gain contours are shown as a function of the normalized source conductance and the normalized available LO power. The range of parameters for which large gain was observed are given by the box.
junctions, the shot noise in the direct detector is small. The noise equivalent power of the square-law detector can be predicted to be $\text{NEP} = 1 \times 10^{-16} \text{ W Hz}^{-1/2}$ for a rapidly modulated signal.

In conclusion, we have observed that SIS heterodyne mixers operated in the quantum limit show large gain with low noise, as well as negative resistance. This behavior is in good agreement with the predictions of quantum mixer theory. We have also improved the performance of the SIS direct detector.

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