Measurement of non-Gaussian shot noise: influence of the environment

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We present the first measurements of the third moment of the voltage fluctuations in a conductor. This technique can provide new and complementary information on the electronic transport in conducting systems. The measurement was performed on non-superconducting tunnel junctions as a function of voltage bias, for various temperatures and bandwidths up to 1GHz. The data demonstrate the significant effect of the electromagnetic environment of the sample. We also present a new setup that allows to measure the frequency dependence of the third moment down to very low temperature.

1 Introduction

Transport studies provide a powerful tool for investigating electronic properties of a conductor. The $I(V)$ characteristic (or the differential resistance $R_{diff} = dV/dI$) contains partial information on the mechanisms responsible for conduction. A much more complete description of transport in the steady state, and further information on the conduction mechanisms, is given by the probability distribution of the current P, which describes both dc current I and the fluctuations $\delta I = I(t) - I$. The current fluctuations can be characterized by the moments of the probability distribution P of order two and higher. However, until now only the second moment has been measured in the many systems studied [1]. In this article we report the first measurements of the third moment of the voltage fluctuations across a conductor, $\langle \delta V^3 \rangle$, where $\delta V = V(t) - V$ represents the voltage fluctuations around the dc voltage V (see also [2]); $\langle . \rangle$ stands for time averaging, or equivalently for averaging over the distribution P. Below we relate $\langle \delta V^3 \rangle$ to $\langle \delta I^3 \rangle$. Our experimental setup is such that the sample is current biased at dc and low frequency but the electromagnetic environment has an impedance $\sim 50\ \Omega$ within the detection bandwidth, 10 MHz to 1.2 GHz. Our results are in agreement with a recent theory that considers the strong effect of the electromagnetic environment of the sample [3]. Moreover, we show that certain of these environmental effects can be dramatically reduced by signal propagation delays from the sample to the amplifier. Finally, we present a new experimental setup which allows to measure the third moment of voltage fluctuations at finite frequency, i.e. with one frequency larger than an interesting energy scale, like the bias voltage or the temperature.

We present the theoretical overview first for the case of voltage bias [1]. In a junction with a low transparency barrier (which corresponds to our samples) biased by a dc voltage V, the current noise spectral density (related to the second moment) is given for low frequency by : $S_{I^2} = eGV \coth(\mathrm{eV}/2\mathrm{k_B T})$ [4] (in A^2/Hz), where e is the electron charge and G is the conductance. Only at high voltage $eV \gg k_B T$ does this reduce to the Poisson result $S_{I^2} = eI$ [4]. The spectral density of the third moment of the current fluctuations in a voltage biased tunnel junction of low transparency is calculated to be: $S_{I^3} = e^2 GV$, independent of temperature [5,6]. By considering how the Fourier components can combine to give a dc signal, we find that $\langle \delta I^3 \rangle = 3S_{I^3}(f_2 - 2f_1)^2$, where the detection bandwidth is from f_1 to f_2. We have experimentally confirmed this unusual dependence on f_1 and f_2 (data not reported here, see [2]).

We next consider the effects of the sample's electromagnetic environment (contacts, leads, amplifier, etc.); the sample is no longer voltage biased. The environment emits noise, inducing fluctuations of the voltage across the sample, which in turn modify the probability distribution

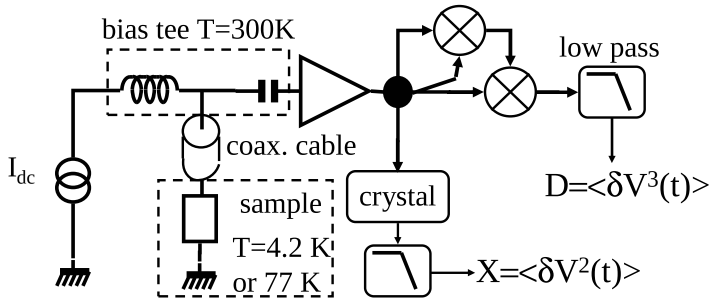

Figure 1: Schematic of the experimental setup.

P. Moreover, due to the finite impedance of the environment, the noise emitted by the sample itself induces also voltage fluctuations. We consider the circuit depicted in the inset of Fig. 2, at first neglecting time delay along the coaxial cable. The noise of the sample of resistance R is modeled by a current generator i. The voltage δV is measured across a resistor R_0, which has a current generator i_0 of noise spectral density $S_{i_0^2}$. One has $\delta V = -R_D(i + i_0)$ with $R_D = RR_0/(R + R_0)$ (R in parallel with R_0). It has been recently predicted that the third moment of P is significantly modified by the environment [3], leading to a spectral density:

$$S_{V^3} = -R_D^3 S_{I^3} + 3R_D^4 S_{i_0^2} \frac{dS_{I^2}}{dV} + 3R_D^4 S_{I^2} \frac{dS_{I^2}}{dV} \quad (1)$$

The first term on the right is like that of the second moment. The negative sign results from an increasing sample current giving a reduced voltage. Our detection method is insensitive to $S_{i_0^3}$. The environment noise i_0 induces voltage fluctuations $\delta V = -R_D i_0$ across the sample. These modify the sample's noise S_{I^2} (which depends on $V(t)$) as $-R_D i_0 dS_{I^2}/dV$, to first order in δV. This is the origin of the second term. The sample's own current fluctuations also modify the sample voltage to contribute similarly, giving the last term of Eq. (1). We present below a simple derivation of how to include the effect of progation time in the coaxial cable, which dramatically affects S_{V^3}.

2 Experimental setup and results

Two samples have been studied. Both are tunnel junctions made of Al/Al oxide/Al, using the double angle evaporation technique [7]. In sample A (made by C. Wilson), the bottom and top Al films are 50 nm thick. The bottom electrode was oxidized for 2 hours in pure O_2 at a pressure of 500 mTorr. The junction area is 15 μm^2. In sample B (made by L. Spietz), the films are 120 nm and 300 nm thick, oxidation was for 10 min, and the junction area is 5.6 μm^2.

We have measured $\delta V(t)^3$ in real time (see Fig. 1). The resistance of the sample is close to 50 Ω, and thus is well matched to the coaxial cable and amplifier. After amplification at room temperature the signal is separated into four equal branches, each of which carries a signal proportionnal to $\delta V(t)$. A mixer multiplies two of the branches, giving $\delta V^2(t)$; a second mixer multiplies this result with another branch. The output of this second mixer, $\delta V^3(t)$, is then low pass filtered, to give a signal which we refer to as D. Ideally D is simply proportional to S_{V^3}, where the constant of proportionality depends on mixer gains and frequency bandwidth. The last branch is connected to a square-law crystal detector, which produces a voltage X proportional to the the rf power it receives: the noise of the sample $\langle \delta V^2 \rangle$ plus the noise of the amplifiers. The dc current I through the sample is swept slowly. We record $D(I)$ and $X(I)$; these are averaged numerically. This detection scheme has the advantage of the large bandwidth it provides ($\sim$ 1 GHz), which is crucial for the measurement. We deduce $S_{V^3} \propto (D(I) - D(-I))/2$.

The magnitude and sign of $\langle \delta V^3 \rangle$ is obtained from measurements of D when the sample is replaced by a programmable function generator.

Sample A was measured at $T = 4.2$K. Its total resistance (tunnel junction + contacts) is 62.6 Ω. The resistance R_A of the junction is extracted from the fit of S_{V^2} as a function of eV/k_BT, with V the voltage drop across the junction. We find $R_A = 49.6$ Ω. R_{diff} is voltage independent to within 1%. The gain of the amplification chain has been calibrated by replacing the sample by a macroscopic 50 Ω resistor whose temperature was varied. We find $\eta = 1$ with a precision of a few percent for both samples. $S_{V^3}(eV/k_BT)$ for $|V| \leq 10$ mV is shown in Fig. 2 (top); these data were averaged for 12 days.

Sample B was measured at $T = 4.2$ K, 77 K and 290 K. The resistance of the junction $R_B = 86\Omega$ is almost temperature independent. The contribution of the contacts is ~ 1 Ω. In Fig. 2 (middle and bottom panels) the averaging time for each trace was 16 hours.

3 Interpretation

To analyze our results, consider again the circuit in the inset of Fig. 2, a simplified equivalent of our setup. $R_0 \sim 50$ Ω is the input impedance of the amplifier, which is connected to the sample through a coaxial cable of impedance R_0 (i.e., matched to the amplifier) . The sample's voltage reflection coefficient is $\Gamma = (R - R_0)/(R + R_0)$. In the analysis we present next we neglect the influence of the contact resistance and impedance mismatch of the amplifier, but we have included it when computing the theory to compare to the data. The voltage $\delta V(t)$ measured by the amplifier at time t arises from three contributions: i) the noise emitted by the amplifier at time t: $R_0 i_0(t)/2$; half of i_0 enters the cable. ii) the noise emitted by the sample (at time $t - \Delta t$, where Δt is the propagation delay along the cable) that couples into the cable: $(1 - \Gamma)Ri(t - \Delta t)/2$; iii) the noise emitted by the amplifier at time $t - 2\Delta t$ that is reflected by the sample: $\Gamma R_0 i_0(t - 2\Delta t)/2$; thus,

$$\delta V(t) = -\frac{R_0}{2}\left[i_0(t) + \Gamma i_0(t - 2\Delta t)\right] - \frac{R}{2}(1 - \Gamma)i(t - \Delta t) \tag{2}$$

For $\Delta t = 0$, Eq. (2) reduces to $\delta V = -R_D(i + i_0)$ with $R_D = RR_0/(R + R_0)$. Thus, $\langle \delta V^3 \rangle = -R_D^3(\langle i^3 \rangle + 3\langle i^2 i_0 \rangle + 3\langle i i_0^2 \rangle + \langle i_0^3 \rangle)$ for $\Delta t = 0$. In this equation the term $\langle i^2 i_0 \rangle$ leads to the second term on the right of Eq. (1). The term $\langle i^3 \rangle$ yields the first term of Eq. (1), and, due to the sample noise modulating its own voltage, the third term of Eq. (1) as well. The terms $\langle i_0^3 \rangle$ and $\langle i i_0^2 \rangle$ are zero. The result for $\Delta t = 0$ corresponds to Eq. (1), which is a particular case of Eq. (12b) of Ref. [3].

The finite propagation time does affect the correlator $\langle i^2 i_0 \rangle$. The term $S_{i_0^2}$ in Eq. (1) has to be replaced by $(\Gamma S_{i_0^2} + S_{i_0(t)i_0(t-2\Delta t)})/(1 + \Gamma)$, where $S_{i_0(t)i_0(t-2\Delta t)}$ is the spectral density corresponding to the correlator $\langle i_0(t)i_0(t - 2\Delta t) \rangle$. For long enough Δt this term vanishes, since $i_0(t)$ and $i_0(t - 2\Delta t)$ are uncorrelated. Thus, the effect of the propagation time is to renormalize the noise temperature of the environment $T_0 = R_0 S_{i_0^2}/(2k_B)$ into $T_0^* = T_0\Gamma/(1 + \Gamma)$.

We now check whether Eq. (1), with $S_{I^3} = e^2 I$ and modified as above to account for finite propagation time, can explain our data. The unknown parameters are the resistance R_0 and the effective environment noise temperature T_0^*. We checked that the impedance of the samples was frequency independent up to 1.2 GHz within 5%. Fig. 2 shows the best fits to the theory, Eq. (1), for all our data. The four curves lead to $R_0 = 42$ Ω, in agreement with the fact that the electromagnetic environment (amplifier, bias tee, coaxial cable, sample holder) was identical for the two samples. We have measured the impedance Z_{env} seen by the sample. Due to impedance mismatch between the amplifier and the cable, there are standing waves along the cable. This causes Z_{env} to be complex with a phase that varies with frequency. We measured that the

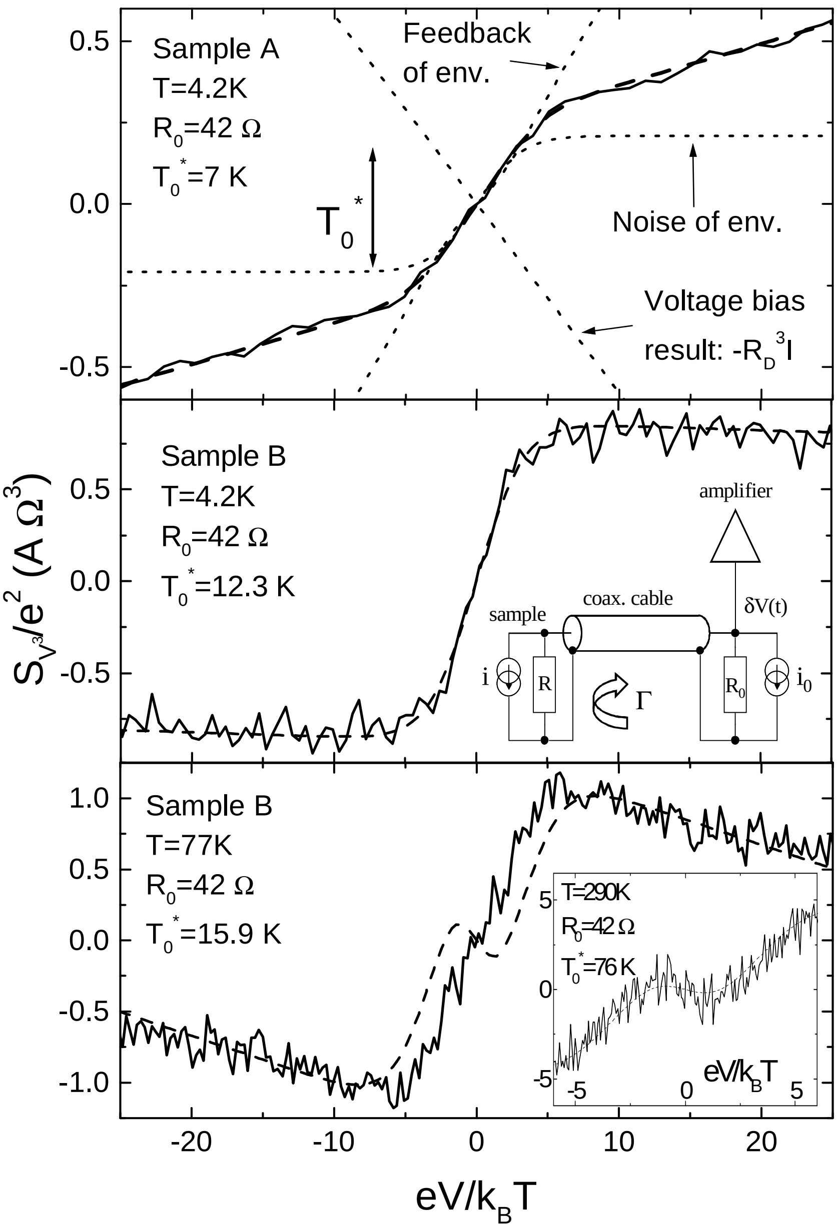

Figure 2: Measurement of $S_{V^3}(eV/k_BT)$ (solid lines). The dashed lines corresponds to the best fit with Eq. (1). The dotted lines in the top plot correspond to the different contributions to S_{V^3} (see text). Inset of the middle plot: schematics of the equivalent circuit used for the theoretical model.

modulus $|Z_{env}|$ varies between 30 Ω and 70 Ω within the detection bandwidth, in reasonable agreement with $R_0 = 42\ \Omega$ extracted from the fits.

We have measured directly the noise emitted by the room temperature amplifier; we find $T_0 \sim 100$ K. The cable of length $\sim$ 2m corresponds to Δt being large for the bandwidth we used. As a consequence, the relevant noise temperature to be used to explain the data is T_0^*. For sample A, $\Gamma = 0.11$; including the contact resistance and cable attenuation one expects $T_0^* = 5$ K ; for sample B, $\Gamma = 0.26$ and one expects $T_0^* = 21$ K. A much shorter cable was used for $T = 290$ K, and the reduction of T_0 is not as significant. These numbers are in reasonably good agreement with the values of T_0^* deduced from the fits (see Fig. 2), and certainly agree with the trend seen for the two samples. Clearly $T_0^* \ll T_0$ for the long cable.

Our data are consistent with a third moment of current fluctuations S_{I^3} being independent of T between 4K and 300K when the sample is voltage biased, as predicted for a tunnel junction. We have also clearly demonstrated the effect of the environment, through its noise and impedance (data not reported here, see [2]). This is of prime importance for designing future measurements on samples with unknown third moment.

4 Towards measurements at finite frequency

In the previous sections we reported measurements in the GHz domain. The use of finite frequency has been chosen in order to achieve good signal-to-noise ratio. However, 1GHz corresponds to almost zero frequency for the sample, even at T=4.2K. It has been recently predicted that S_{I^3} at finite frequency has interesting properties: i) In any system, one might expect a crossover from classical to quantum noise at frequencies f of the order of the bias voltage and/or the temperature, as is seen for S_{I^2} [8]. Contrary to the second moment, the third moment vanishes at zero voltage, and thus has no voltage-independent contribution (coming from vacuum fluctuations). Since the voltage-dependent part of S_{I^2} vanishes at $hf \gg eV, k_BT$ one might expect the same result for S_{I^3}. In contrast, the prediction is that S_{I^3} for a tunnel junction is frequency independent [9]. ii) In systems with an intrinsic dynamic characterized by a time scale τ_D (e.g., the dwell time of a cavity, the diffusion time of a disordered wire), a dispersion in S_{I^3} at $f \sim \tau_D^{-1}$ has been predicted, which does not appear in S_{I^2} [10]. iii) The effect of the environment in the quantum regime has to be revisited; the distinction between absorption/emission of photons might be relevant. iv) The definition of the third moment in terms of quantum operators raises the problem of their time ordering [5]. A crossover from the Keldysh ordered to the fully symmetric version of S_{I^3} might occur at finite frequency [11].

Introducing the Fourier transform $i(f)$ of $\delta I(t)$, and disregarding the problems of ordering the current operators , one has $S_{I^3}(f, f') = \langle i(f)i(f')i(-f-f')\rangle$. The setup of Fig. 1 corresponds to integrating the frequencies f and f' between f_1 and f_2. This setup cannot be used to measure e.g. $S_{I^3}(0, f)$ due to the broadband integration. In order to perform such a measurement we have constructed the setup of Fig. 3a. The signal $\delta V(t)$ is split into two frequency bands, LF=$]0, f_3]$ and HF=$[f_1, f_2]$. The voltage is squared in the HF band (left branch of Fig. 3a) with a high speed tunnel diode, then low-pass filtered with a cutoff f_3. Thus one has products of the form $i(f)i(-f')$ at the end of the HF branch. This result is multiplied by a mixer to the LF branch, then low-pass filtered to get a dc signal. This signal corresponds to $S_{V^3}(0, \bar{f})$ if $(f_2 - f_1)$ and f_3 are small enough, with $\bar{f} \sim (f_1 + f_2)/2$. f_1 and f_2 can be varied by changing the various fiters.

The measurement of sample A at $T = 4.2$K and $T = 50$mK with the new setup operating in the LF=10-200MHz and HF=1-2.4GHz bandwidths is presented in Fig. 3b. We find subtle but interpretable new results. The fit of these data with eq. (1) leads to $R_0 = 40\Omega$ and $T_0^*(T = 4.2K) = -0.4K$. The slope of S_{V^3} at high voltage is found to be temperature independent, like it is between 300K and 4.2K (see Fig. 2). The negative T_0^* comes from the Johnson noise of

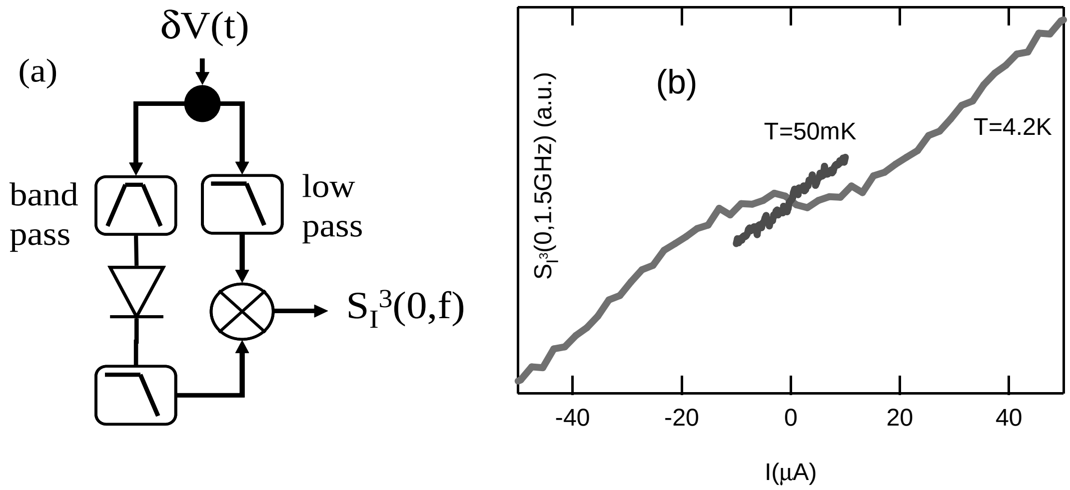

Figure 3: (a) Setup for the measurement of $S_{V^3}(0, \bar{f})$. The sample, bias tee and amplifiers have been omitted, see Fig. 1. (b) Measurement of $S_{V^3}(0, \bar{f} \sim 1.5\text{GHz})$ on sample A at $T = 4.2$K and $T = 50$mK.

the 12Ω contact resistance. The current fluctuations emitted by the contact result in currents of opposite signs running through the sample and the amplifier. As a consequence, the contact contributes to T_0^* with a negative sign. Since we use in this new setup a cryogenic amplifier with low noise temperature T_0, and since the Johnson noise of the contact is not affected by the propagation time, its contribution dominates T_0^* at 4.2K. We indeed observe a sign reversal of T_0^* when cooling the sample below 1K, since the noise of the amplifier dominates at low enough temperature (see Fig. 3b). The non-linear behavior at low voltage at T=4.2K is similar to the one observed at room temperature with the previous setup, see Fig. 2, i.e. when the noise emitted by the sample is larger than the noise emitted by the amplifier, revealing the contribution of the feedback of the environment. The study of the regime $hf > eV$ requires the use of a higher frequency bandpass filter in the HF branch, and is in progress.

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