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Effect of spin-orbit and spin-flip scattering on conductance fluctuations

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We discuss the effect of the spin-dependent scattering on conductance fluctuations in the regime where the spin-orbit scattering length l_{so} and the magnetic spin-flip scattering length l_S are comparable to the electron-phase-breaking length l_ϕ. In addition to affecting the magnitude of the fluctuations, we find that the presence of spin-dependent scattering changes the characteristic magnetic-field scale of fluctuations. Our results provide a simple way of interpreting experimental results for any sample geometry.

Quantum coherence of electrons in disordered systems has been studied[1-3] extensively over the past few years. Among the various manifestations of this coherence, conductance fluctuations capture the essence of the physics involved. Due to the electron spin, the magnitude of the conductance fluctuations is expected to be affected by spin-orbit scattering, Zeeman splitting, and spin-flip scattering. There have been specific predictions for the magnitude of conductance fluctuations in the presence of these perturbations. It has been shown[4-8] that in the presence of strong spin-orbit scattering, the magnitude of the mean-square fluctuations is reduced by a factor of 4. Feng[8] has derived a relatively complicated expression for the conductance fluctuations in the presence of spin-orbit scattering. We shall show below that his expression can be simplified substantially when written in terms of the singlet and triplet contributions, similar to the weak-localization calculations.[9,10] This enables us to calculate the characteristic magnetic field scale of the fluctuations, which we find is affected by the presence of moderate spin-orbit scattering. Our results should be particularly useful to experimentalists who wish to analyze data on conductance fluctuations in samples with moderate spin-orbit scattering. We shall also discuss the effect of scattering of electrons from magnetic impurities.

In order to calculate the fluctuations, we have to evaluate[2] diagrams involving two conductivity loops [Fig. 1(b)]. In the absence of spin-dependent scattering, the contribution of these diagrams, including both diffusion (particle-hole) and Cooper (particle-particle) channels, can be written as

$$\langle \delta G_0^2(l_\phi)\rangle \sim \left[\frac{e^2}{h}\right]^2 \left[\frac{1}{L^{4-d}}\right] \left[\sum_q \frac{1}{(q^2+l_\phi^{-2})^2}\right]. \qquad (1)$$

Here l_ϕ is the phase-breaking length, L is the length of the sample in d dimensions, and q is the eigenvalue of the momentum operator $\hat{\mathbf{q}}$ for the geometry under consideration. The term with the summation over q in Eq. (1) is the contribution of the two diffusion (or Cooper) impurity ladders $\Lambda(q)$ shown in Fig. 1(b). To evaluate the effect of spin-dependent scattering, we should consider the explicit spin dependence of these impurity ladders. The calculation is similar to the one for weak localization,[9] except that for conductance fluctuations, one does not allow magnetic scattering to connect the particle and hole (or particle and particle) lines of the ladder. This is because they belong to two different conductivity loops which represent two different measurements, and we do not expect the magnetic impurity spin to remain static between measurements.[2] Normal elastic impurity scattering and spin-orbit scattering are allowed to connect the two loops, since we do not expect the corresponding potentials to change between the measurements.

In what follows, we shall consider only the diffusion channel in detail. The calculation for the Cooper channel is the same, except for some minor differences. The total scattering potential is[9,10]

$$U_{ss'}(\mathbf{k},\mathbf{k}') = U + U_s \mathbf{S}\cdot\boldsymbol{\sigma}_{ss'} + iU_{so}(\hat{\mathbf{k}}\times\hat{\mathbf{k}}')\cdot\boldsymbol{\sigma}_{ss'}. \qquad (2)$$

The first term is the elastic-scattering potential, the

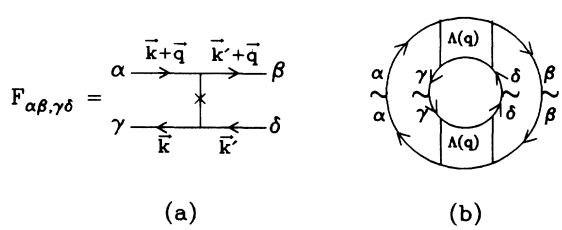

FIG. 1. Examples of diagrams used to calculate conductance fluctuations: (a) a part of the diffusion (particle-hole) impurity ladder; (b) one of the corresponding diagrams in the diffusion channel for the calculation of conductance fluctuations. The diagram for the particle-particle (Cooper) channel would have the arrows in the two loops pointing in the same direction.

second the potential due to the impurity spin **S**, and the third the spin-orbit potential. $\sigma_{ss'}$ refers to the Pauli spin matrix of the electron, $\hat{\mathbf{k}}$ is the wave vector of the incident electron, and $\hat{\mathbf{k}}'$ the wave vector of the scattered electron. The total scattering rate is given by[9,10]

$$\tau^{-1}=2\pi N(0)\left[U^2+U_{so}^2\sum_i|(\hat{\mathbf{k}}\times\hat{\mathbf{k}}')_i|^2+U_s^2\sum_i\langle S_i^2\rangle\right]$$
$$=\tau_0^{-1}+\tau_{so}^{-1}+\tau_s^{-1}. \quad (3)$$

$N(0)$ is the electron density of states at the Fermi energy. A single part of the particle-hole impurity ladder is shown in Fig. 1(a). The cross denotes the scattering potential without magnetic-impurity scattering. The diagram can be evaluated to give

$$F_{\alpha\beta,\gamma\delta}=[2\pi N(0)]^{-1}[\tau_0^{-1}\delta_{\alpha\beta}\delta_{\delta\gamma}+\tfrac{1}{3}(\tau_{so}^{-1})\sigma_{\alpha\beta}\sigma_{\delta\gamma}], \quad (4)$$

where we have assumed isotropic scattering. [The factor $\frac{1}{3}$ arises because one does not sum over the directions i of Eq. (3).] The impurity diffusion ladder is given by a series of diagrams such as Fig. 1(a). The series can be summed, and the result, in matrix form, is given by

$$\Lambda(q)=F+2\pi N(0)F\tau(1-\tau/\tau_\phi-Dq^2\tau)\Lambda(q), \quad (5)$$

where D is the diffusion constant and F and $\Lambda(q)$ are now 4×4 matrices. The phase-relaxation time τ_ϕ determines the characteristic time of decay of the particle-hole Green's function in the absence of spin-dependent mechanisms. Equation (5) can be rewritten as

$$2\pi N(0)\tau\Lambda(q)=[F'^{-1}-(1-\tau/\tau_\phi-Dq^2\tau)I]^{-1}, \quad (6)$$

where $F'=2\pi N(0)\tau F$ and I is the unit matrix. Now, since $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=-2(J^2-s_1^2-s_2^2)$, where $\mathbf{s}_{1,2}=\frac{1}{2}\boldsymbol{\sigma}_{1,2}$ and $\mathbf{J}=\mathbf{s}_1-\mathbf{s}_2$ (the minus sign is because one of the σ's refers to a hole), F' can be written in the J^2, s_1^2,s_2^2 representation as

$$F'=(\tau/\tau_0+\tau/\tau_{so})I-\tfrac{2}{3}(\tau/\tau_{so})J^2 \quad (7)$$

so that the singlet and triplet matrix elements are given by

$$F_1'=(\tau/\tau_0+\tau/\tau_{so})=1-(\tau/\tau_s), \quad (8a)$$

$$F_2'=1-(\tau/\tau_s)-\tfrac{4}{3}(\tau/\tau_{so}). \quad (8b)$$

The corresponding matrix elements of $\Lambda(q)$ are obtained from

$$\lambda_1=[2\pi N(0)\tau^2]^{-1}[Dq^2+(\tau_\phi^{-1})+(\tau_s^{-1})]^{-1}, \quad (9a)$$

$$\lambda_2=[2\pi N(0)\tau^2]^{-1}[Dq^2+(\tau_\phi^{-1})+(\tau_s^{-1})+\tfrac{4}{3}(\tau_{so}^{-1})]^{-1}. \quad (9b)$$

The diagrams for the conductance fluctuations involve products of two diffusion ladders (or two Cooper ladders). To see which matrix elements are allowed by spin conservation, we should transform the matrix Λ into the s_1,s_{1z}, s_2,s_{2z} representation. In this representation,

$$\Lambda=\begin{vmatrix}\frac{(\lambda_1+\lambda_2)}{2} & 0 & 0 & \frac{(\lambda_1-\lambda_2)}{2}\\ 0 & \lambda_2 & 0 & 0\\ 0 & 0 & \lambda_2 & 0\\ \frac{(\lambda_1-\lambda_2)}{2} & 0 & 0 & \frac{(\lambda_1+\lambda_2)}{2}\end{vmatrix}.$$

Let us label spin indices of the top diffusion ladder by $\langle\alpha\beta\gamma\delta\rangle$, as shown in Fig. 1(b). Then, the only indices of the bottom ladder allowed by spin conservation are $\langle\gamma\delta\alpha\beta\rangle$. The diagrams for the conductance fluctuations involve products of the form $\langle\alpha\beta\gamma\delta\rangle\times\langle\gamma\delta\alpha\beta\rangle$. The only six nonzero elements of the matrix Λ give rise to six terms, the sum of which is $3(\lambda_2)^2+(\lambda_1)^2$. From Eq. (9), the appropriate length scales for λ_1 and λ_2 are

$$\lambda_1:\ l_1^{-2}=l_\phi^{-2}+l_s^{-2}, \quad (10a)$$

$$\lambda_2:\ l_2^{-2}=l_\phi^{-2}+l_s^{-2}+\tfrac{4}{3}l_{so}^{-2}. \quad (10b)$$

Here as usual, $l_\phi=\sqrt{D\tau_\phi}$, $l_{so}=\sqrt{D\tau_{so}}$, and $l_s=\sqrt{D\tau_s}$. The Cooper channel contribution is also split into singlet and triplet parts, with the length dependence of the singlet term given by l_1 and that of the triplet term by l_2.

In the absence of spin-orbit and spin-flip scattering, $\lambda_1=\lambda_2=\lambda_0\equiv[2\pi N(0)\tau^2]^{-1}[Dq^2+(\tau_\phi^{-1})]^{-1}$. From Eq. (1), the mean-square amplitude of the conductance fluctuations is given by

$$\langle\delta G_0^2(l_\phi)\rangle\simeq\left(\frac{e^2}{h}\right)^2\frac{[2\pi N(0)D\tau^2]^2}{L^{4-d}}\sum_q\lambda_0^2. \quad (11)$$

With spin-orbit and spin-flip scattering, the mean-square amplitude is given by

$$\langle\delta G^2\rangle\simeq\left(\frac{e^2}{h}\right)^2\frac{[2\pi N(0)D\tau^2]^2}{L^{4-d}}\left(\tfrac{3}{4}\sum_q\lambda_2^2+\tfrac{1}{4}\sum_q\lambda_1^2\right). \quad (12)$$

From Eq. (11), it is clear that this can be expressed as

$$\langle\delta G^2\rangle=\tfrac{3}{4}\langle\delta G_0^2(l_2)\rangle+\tfrac{1}{4}\langle\delta G_0^2(l_1)\rangle, \quad (13)$$

where $\langle\delta G_0^2(l_1)\rangle$ and $\langle\delta G_0^2(l_2)\rangle$ are given by Eq. (1), with l_ϕ replaced by l_1 and l_2. Thus, the expression for the mean-square amplitude of the conductance fluctuations in the presence of spin-dependent scattering can be obtained from the expression in the absence of spin-dependent scattering merely by putting the appropriate singlet and triplet length scales l_1 and l_2 in place of l_ϕ. This result is valid for *any* experimental configuration. Equation (13) yields the correct limiting behavior predicted in Refs. 4–8, i.e., strong magnetic-impurity scattering destroys the conductance fluctuations, whereas strong spin-orbit scattering will reduce the amplitude by one half.

In experiments on weak localization, one can determine l_ϕ, l_{so}, and l_s by fitting the weak localization magnetoconductance to the full spin-dependent theory.[9] For the conductance fluctuations, the quantity analogous to the weak-localization magnetoconductance is the magnetic-field autocorrelation function,

$$F(\Delta B)=\langle\delta G(B)\delta G(B+\Delta B)\rangle,$$

where the angle brackets denote an average over the magnetic field B. In the presence of spin-dependent scattering, this function can also be split into singlet and triplet contributions

$$F(\Delta B)=\tfrac{3}{4}F_0(\Delta B,l_2)+\tfrac{1}{4}F_0(\Delta B,l_1), \quad (14)$$

where $F_0(\Delta B,l_\phi)$ is the autocorrelation function in the ab-

sence of spin-dependent scattering. Thus, if the form of $F_0(\Delta B, l_\phi)$ is known, one should be able to determine the electron scattering lengths l_ϕ, l_{so}, and l_s by fitting the experimental autocorrelation function to Eq. (14). In passing, we note that $F_0(\Delta B, l_\phi)$ at low magnetic fields is not independent of the absolute magnetic field B, because of the magnetic-field dependence of the particle-particle impurity ladder.[3] In addition, l_1 and l_2 may themselves be field dependent, through the field dependence of the magnetic impurity scattering. Both these effects give rise to a complicated magnetic-field dependence at low magnetic fields. Thus, in order to facilitate comparison with theory, it is better to measure the autocorrelation function at high magnetic fields.

To illustrate the practical utility of these calculations, we consider the specific example of a long one-dimensional wire in the absence of energy averaging and magnetic-impurity scattering. For this case Eq. (1) yields[2]

$$\langle \delta G_0^2(l_\phi) \rangle \sim \left(\frac{e^2}{h} \right)^2 \left(\frac{l_\phi}{L} \right)^3 . \tag{15}$$

Figure 2 shows the amplitude of the fluctuations as a function of l_ϕ from Eqs. (13) and (15) for $l_{so}=0.5$ μm. As expected, the amplitude of the fluctuations for $l_\phi \gg l_{so}$ is half that for $l_\phi \ll l_{so}$. However, it is interesting to note that the crossover from the weak spin-orbit-scattering limit to the large spin-orbit-scattering limit is rather rapid: for $l_\phi \simeq 3 l_{so}$, we are already in the strong spin-orbit-scattering limit.

The field dependence of the fluctuations can be characterized by the half-width B_c of the autocorrelation function $F(\Delta B)$, defined by $F(B_c)=\frac{1}{2}F(0)$. For a wire of width W, in the absence of spin-dependent scattering, B_c is given by[11] $B_{co}=0.42\Phi_0/(l_\phi W)$. In the presence of spin-orbit scattering, B_c depends on the relative magnitudes of l_ϕ, l_s, and l_{so}. Figure 3 shows the ratio of B_c to B_{co} as a function of l_ϕ/l_{so}, for a long wire with the same parameters as in Fig. 2. In both the large and small spin-orbit-scattering limits, B_c approaches B_{co}. In the intermediate regime, however, there is an enhancement of B_c, with a maximum of $\simeq 1.22$ at $l_\phi/l_{so} \simeq 0.5$. This means that, when $l_\phi \simeq l_{so}$, one cannot determine the phase coherence length l_ϕ by measuring B_c alone. To determine l_ϕ, it is necessary to fit the measured field autocorrelation function to Eq. (14), using l_ϕ, l_s, and l_{so} as fitting parameters.

Experiments have been performed by Millo *et al.*[12] on two-dimensional $GaAs/Al_xGa_{1-x}As$ heterojunctions to study the effect of spin-orbit scattering on conductance fluctuations. Below 2 K, these samples show weak antilocalization at very low magnetic fields, indicating the presence of spin-orbit scattering. Millo *et al.* observe a decrease in the amplitude of the fluctuations at low temperatures from that expected by extrapolating the data from higher temperatures. They attribute this decrease in amplitude to the effect of spin-orbit scattering. They also observe an increase in the correlation field B_c at low temperatures, over the value expected from extrapolating the high-temperature data. In our description, this is due to the fact that, as l_ϕ becomes comparable to l_{so} at low temperatures, the correlation field increases, as shown in Fig. 3.

In conclusion, we find that the presence of spin-dependent scattering affects both the amplitude and the characteristic magnetic-field scale of the conductance fluctuations. These effects can be quantitatively described with simple modifications to the existing theory, facilitating comparison with experiment.

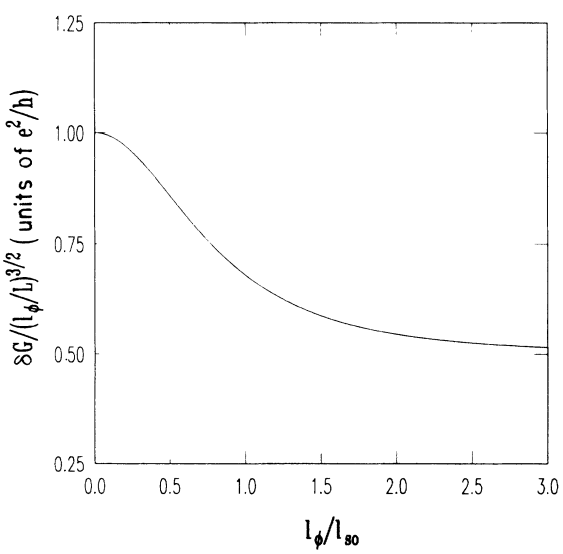

FIG. 2. Normalized amplitude of the conductance fluctuations for a one-dimensional wire of length L as a function of the ratio l_ϕ/l_{so}, from Eqs. (13) and (15), in the absence of energy averaging and magnetic-impurity scattering. l_{so} is 0.5 μm.

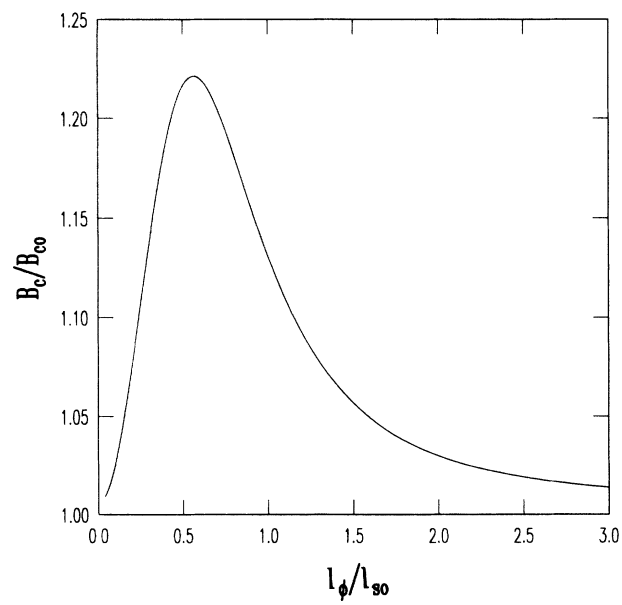

FIG. 3. B_c/B_{co} as a function of l_ϕ/l_{so} for the same case shown in Fig. 2. B_{co} is defined as $B_{co}=0.42\Phi_0/(l_\phi W)$, and the width of the wire is 0.1 μm.

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[1]S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).

[2]P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).

[3]B. L. Altshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].

[4]B. L. Altshuler and B. I. Shkovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)].

[5]A. D. Stone, Phys. Rev. B **39**, 10736 (1989).

[6]Yigal Meir, Y. Gefen, and Ora Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1989).

[7]N. Zanon and J.-L. Pichard, J. Phys. (Paris) **49**, 907 (1988).

[8]Shechao Feng, Phys. Rev. B **39**, 8722 (1989).

[9]S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).

[10]G. Bergmann, Phys. Rep. **107**, 1 (1984).

[11]C. W. J. Beenakker and H. van Houten, Phys. Rev. B **37**, 6544 (1988).

[12]O. Millo, S. J. Klepper, M. W. Keller, D. E. Prober, and R. N. Sacks, Bull. Am. Phys. Soc. **35**, 731 (1990); and (unpublished).