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Magnetotransport in a chaotic scattering cavity with tunable electron density

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Abstract

We have measured the resistance versus magnetic field and Fermi energy for two stadium-shaped cavities in which chaotic scattering is expected. We fit the power spectra of the magnetoresistance fluctuations to semiclassical chaotic scattering theory and find a characteristic field which scales with cavity dimensions as predicted by classical simulations. Measurements of conductance versus Fermi energy indicate a significant contribution from short, non-chaotic paths. We have also studied the energy-averaged conductance as a function of magnetic field. We observe a ballistic weak localization effect similar to that predicted by recent calculations.

1. Introduction

The 2D electron gas of a clean GaAs/AlGaAs heterostructure can be confined to a cavity small enough that large angle elastic scattering is dominated by the walls rather than random impurities. At temperatures of about 0.1 K, inelastic scattering is weak enough that electrons can traverse a 1 μm cavity several times while maintaining phase coherence. The electron wavelength is still much smaller than the cavity, however, so that electron transport may be described semiclassically. Such a cavity is thus useful in testing the predictions of the semiclassical theory of chaotic scattering [1,2]. This theory predicts random fluctuations in the resistance of a chaotic cavity such as a stadium when a perpendicular magnetic field is applied or when the electron density is changed. There is also a recent prediction for a weak localization effect which reflects the chaotic nature of the cavity [3]. The observation of a difference between magnetoresistance fluctuations in nominally chaotic and non-chaotic cavities has been reported previously [4]. In this paper we report the observation of the scaling of magnetoresistance fluctuations with cavity size. We also find evidence for an important contribution from short, non-chaotic paths. Finally, we study the weak localization in the cavity as a function of Fermi energy and find an energy-averaged effect in agreement with recent numerical studies [3].

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2. Fabrication and measurement

Our samples are fabricated from GaAs/Al GaAs heterostructures having bulk mobility of 60 $m^2/V \cdot s$ and density of 2.6×10^{15} m^{-2}. After fabrication the mobility of a small cavity is difficult to assess, but the density is found to be similar to the bulk. Confinement of the electron gas is achieved by patterning a Ti/Au metal mask with electron-beam lithography and exposing it to low energy ($\sim$ 200 eV) Xe ions [5], which makes the unprotected areas insulating. The metal mask is a self-aligned Schottky barrier gate which allows the density to be varied over a small range without significantly changing the shape of the cavity.

The data we present here are from a sample with two cavities whose shape and dimensions are given in the insets of Fig. 2. The offset lead geometry was chosen to eliminate direct paths which do not interact with the edges of the cavity. The change in electron density with gate voltage is calibrated from the change in one or more minima of the Shubnikov–de Haas oscillations. We estimate an electron density of $(2.1 + 2.3V_g) \times 10^{15}$ m^{-2}, where V_g is the gate voltage in volts, over the range $|V_g| \leq 0.1$ V. The sample is mounted in a dilution refrigerator and all data presented here are measured with the mixing chamber at 50 mK. The actual electron temperature is estimated to be 100 mK or slightly higher [6]. The voltage drop across the cavity is always less than 10 μV.

3. Magnetoresistance fluctuations

Fig. 1 shows the magnetoresistance of the two stadia measured with $V_g = -0.02$ V. Both stadia show random, reproducible fluctuations but on different magnetic field scales. The data are not symmetric about $B = 0$ because we make a true four probe measurement. We have interchanged current and voltage leads and find that the Onsager relation for four probe measurements [7] is satisfied.

The theory of quantum chaotic scattering pre-

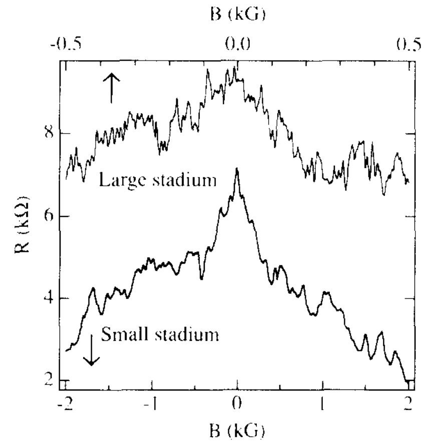

Fig. 1. Magnetoresistance of the small and large stadia with $V_g = -0.02$ V, $T \approx 100$ mK.

dicts the following form for the power spectrum $S(f)$ of the fluctuations [1]:

$$S(f) = S(0)(1 + 2\pi\phi_0\alpha f)\, e^{-2\pi\phi_0\alpha f}, \qquad (2)$$

where f is inverse magnetic field, $\phi_0 = h/e$ is the flux quantum, and α is a parameter that characterizes the chaotic dynamics of classical particles scattering in a cavity of the same shape. Specifically, α determines the probability P that a classical particle will enclose an area A before escaping the cavity according to

$$P(|A|) \propto e^{-2\pi\alpha|A|}. \qquad (3)$$

The power spectra of the data in Fig. 1 are shown in Fig. 2 [8]. The smooth lines are fits to (2) for $0.006 \leq f \leq 0.022$ G^{-1} (small stadium) and for $0.024 \leq f \leq 0.184$ G^{-1} (large stadium). The lower bound corresponds to an area of about $(2\pi\alpha)^{-1}$, which is approximately the smallest area for which (3) is expected to hold [1]. The upper bound is the largest f for which (2) remains a good fit to the data (found iteratively). It is always larger than $(\phi_0\alpha)^{-1}$. The fits give $\phi_0\alpha = 61.3$ G (small stadium) and 8.5 G (large stadium). The frequency interval used in the fit affects the value of $\phi_0\alpha$ by as much as $\pm 20\%$. Fig. 3 shows the range of $\phi_0\alpha$ values which give reasonable fits over some frequency interval, for measurements at several values of V_g. The bar at $V_g = 0$ represents several measurements separated by thermal cy-

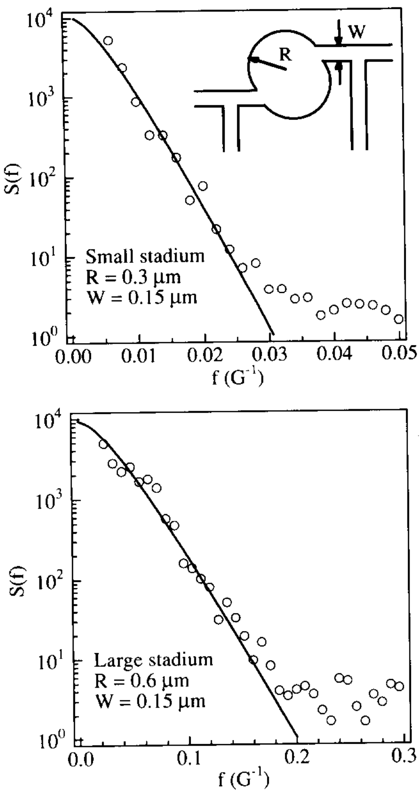

Fig. 2. Power spectra of the data in Fig. 1 (circles) and fits to Eq. (2) (lines). Insets: Stadium shape with leads and dimensions for each stadium.

cles to $T \geq 77$ K. There is no apparent trend in $\phi_0\alpha$ with V_g, which indicates the size and shape of the cavity are independent of V_g over the

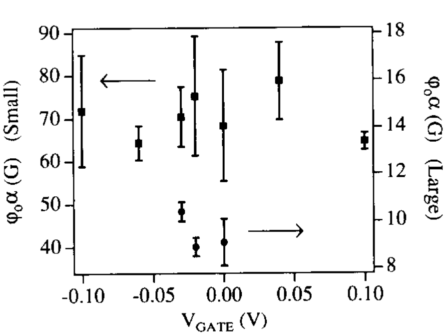

Fig. 3. Characteristic field $\phi_0\alpha$ of the small (squares) and large (circles) stadia for several gate voltages. The bars give the range of values which give good fits and the points are the mean values.

Table 1
Characteristic field $\phi_0\alpha$ for both stadia

Stadium	$\phi_0\alpha$ (G)	
	Simulation	Experiment
Small	23	70
Large	3.5	9.3

Simulation values are from Ref. [9]. Experimental values are average values for several thermal cycles and gate voltages.

range shown. The mean $\phi_0\alpha$ for each stadium over all V_g is shown in Table 1.

The expected value of $\phi_0\alpha$ for a stadium can be found by numerical simulation of classical particles scattering in the cavity [1]. Our stadium shape with offset leads has not been simulated, but simulations for symmetric stadia with directly opposite leads may be used as a first estimate. For symmetric stadia, simulations [9] give the values of $\phi_0\alpha$ shown in Table 1. The experimental value of $\phi_0\alpha$ for each stadium is about three times larger than the simulation value, indicating the typical enclosed areas in our stadia are about three times smaller than expected. Jensen [2] has argued that a tendency for trajectories to circulate in one direction before being backscattered exists in the symmetric stadium with $R \gg W$. This effect increases the typical area enclosed by a factor of about four. Our experimental results indicate a much weaker circulation effect, which is likely due to the cavity edges not being perfectly smooth. A more realistic simulation of our shape would be helpful. It is important to note that the ratio of the two experimental values agrees fairly well with the prediction from the simulations, indicating that the fluctuations scale with cavity size as expected. Also note that the ratio is not simply the inverse ratio of the total cavity areas. Jensen [2] has derived an analytical expression for $\phi_0\alpha$ in terms of R and W for symmetric stadia. His result predicts values of 18 G for the small stadium and 3.8 G for the large stadium.

4. Conductance versus wavevector

Fig. 4 shows the conductance of the small stadium as a function of Fermi wavevector at

$B = 0$. Again we observe random but reproducible fluctuations. These fluctuations are also expected to have a power spectrum predicted by semiclassical theory [1]. This aspect of the data is still under investigation and will be discussed elsewhere.

Here we focus on the contribution of short, non-chaotic paths to the conductance. Two such paths are suggested in the inset of Fig. 4. Path 1 goes directly between the two leads and path 2 bounces once off each side of the cavity. In the semiclassical picture, the transmission of these two paths has a phase factor $\exp[ik_F(L_1 - L_2)]$, where L_1 and L_2 are the path lengths. This phase will oscillate as k_F is changed, and if such a pair of paths contributes strongly to the conductance, the Fourier power of $G(k_F)$ will show a peak at the frequency of the oscillation, $1/\Delta k_F = (L_1 - L_2)/2\pi$. The (unaveraged) Fourier power of $G(k_F)$ for the small stadium at several values of B is shown in Fig. 5. At $B = 0$ there are two peaks present, indicating two pairs of paths make an important contribution. At $B = 20$, 35, and 50 G (not shown), the same two peaks are found with nearly the same amplitudes. At $B = 70$ G, the higher frequency peak begins to disappear and a new peak appears at low frequency. The lower frequency peaks persist until $B = 350$ G, the largest field used. The magnetic field causes the paths to curve, and a path will cease to contribute strongly to the conductance when it no longer enters the opposite lead. A higher frequency peak presumably involves a longer path, and thus will be affected by a smaller field, consistent with our observations.

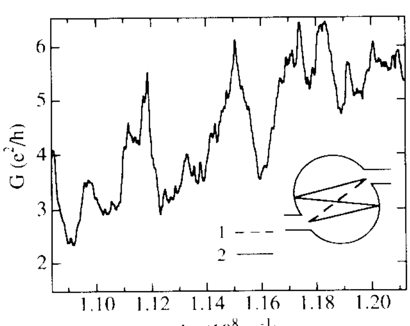

Fig. 4. $G(k_F)$ for the small stadium at $B = 0$. The wavevector range shown corresponds to $-0.1 \le V_g \le 0.1$ V. The number of modes in the leads, $k_F W/\pi$, changes from 5.2 to 5.8 over this range. Inset: A possible pair of short paths inside the cavity (see text).

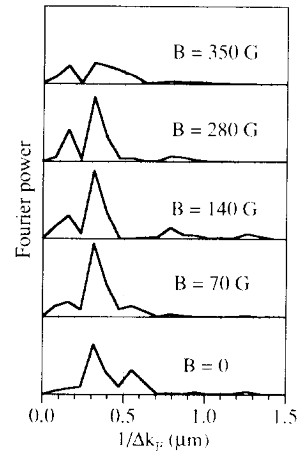

Fig. 5. Fourier power of $G(k_F)$ for the small stadium, after subtraction of a linear background from $G(k_F)$.

At this point we cannot identify the exact paths which are responsible for the observed peaks in the Fourier spectra. It would be helpful to have similar data for a range of cavity sizes. Unfortunately, the ability to vary the density in the large stadium was lost before such data could be obtained. Nonetheless, our results show that some pairs of relatively short paths are important and that the longer paths which reflect the chaotic classical dynamics of the cavity do not completely determine the conductance [10]. This was mentioned in reports of numerical studies [1], but has not been measured previously.

5. Weak localization

The data for the small stadium in Fig. 1 show a pronounced peak at $B = 0$. This feature has a width comparable to $\phi_0 \alpha$, and was identified previously [4] as a weak localization (WL) effect due to breaking of time reversal symmetry by the magnetic field. We do not find a strong peak at $B = 0$ for all values of V_g, nor after every thermal cycle to $T \ge 77$ K for $V_g = 0$. WL always produces negative magnetoresistance, but when fluctua-

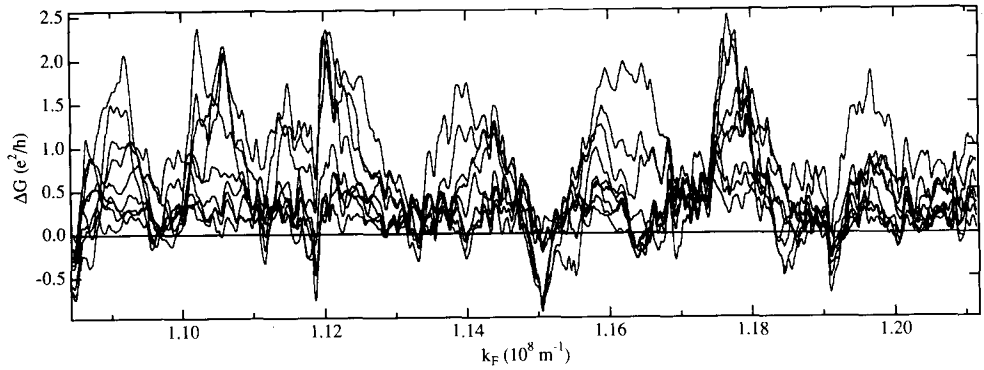

Fig. 6. $\Delta G(k_F, B) \equiv G(k_F, B) - G(k_F, 0)$ for the small stadium at B = 20, 35, 50, 70, 100, 140, 210, 280, and 350 G. The traces are generally in order of increasing B from bottom to top.

tions are also present, the net magnetoresistance can be of either sign. The WL can only be distinguished from the fluctuations by performing an ensemble average. In our sample the gate allows us to average over Fermi energy.

The WL can be expressed as the change in conductance $\Delta G(k_F,B) \equiv G(k_F,B) - G(k_F,0)$. This is shown in Fig. 6 for several values of B. This plot explicitly shows the variation in the WL effect as we access different members of the ensemble of the cavity by changing k_F. For some members the effect is large and for others it is small, and both positive and negative effects are seen. The energy-averaged WL effect is simply the mean of ΔG over the entire range of Fig. 6, and it is shown in Fig. 7. We find $\langle \Delta G \rangle$ increases sharply for small B and then rises more slowly for

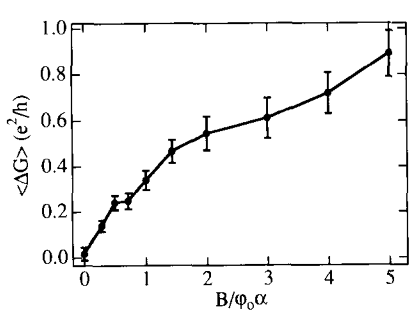

Fig. 7. Mean ΔG for the wavevector range shown in Fig. 6. The field scale has been normalized using $\phi_0\alpha = 70$ G. The point at $B = 0$ is the difference between two $G(k_F)$ traces at $B = 0$ taken before and after all the other traces (three days apart).

B larger than about $1.5\phi_0\alpha$. In Ref. [3], the energy-averaged change in conductance was calculated from numerically-generated $G(k_F)$ traces in the same fashion as described here. Very similar behavior was found for $\langle \Delta G \rangle$ versus B in stadium-like structures with no direct paths between the leads.

Ref. [3] also explored the effect of cavity shape on the weak localization effect. It was found that the effect is qualitatively different in chaotic and non-chaotic cavities, and that the symmetry of the cavity affects the size of the effect by a substantial amount. Ongoing studies are aimed at testing these predictions.

6. Conclusion

We have observed fluctuations in the resistance of chaotic cavities as a function of magnetic field which scale with the cavity size as expected from simulations of classical particles. There appears to be a weaker circulation effect in our cavities than is seen in simulations of shapes with symmetric leads and perfectly smooth edges. We find peaks in the power spectrum of $G(k_F)$ which indicate an important contribution from pairs of short, non-chaotic paths. We have measured weak localization as a function of energy and we observe an energy-averaged weak localization effect similar to the effect seen in recent calculations [3].

7. Acknowledgements

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8. References

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[10] An argument has been made which predicts peaks in the Fourier spectrum related to unstable periodic orbits in chaotic phase space [2]. These orbits, however, typically do not connect one cavity lead to the other and will not contribute strongly to the conductance. Thus we believe it is unlikely they will be reflected in the Fourier spectra.